**Hyperbola**

1. Prove that the point lies on the hyperbola .

 Another point on this hyperbola is given by .

 Find the equation of chord .

 Deduce that the equation of the tangent to the hyperbola at is given by

 This tangent intersects the x-axis at the point A and the y-axis at the point B.

 Find the area of in terms of p.

 Therefore the point lies on the curve .

 Gradient of the chord =m =

 The equation of chord :

 The chord becomes the tangent to the curve at when Q = P, that is, q = p.

 This tangent intersects the x-axis when or

 and intersects the y-axis when or

 Area of

**2.** Given four points , . A point P moves so that its distances are related by the equation: .

 Show that the locus of P is a hyperbola and find the equations of its asymptotes.

 Let ,

 The locus of P is a hyperbola:

 The asymptotes of the hyperbola are or .

**3.** **(a)** A curve of the form has asymptotes and passes through the point . Find the equation of the this curve in terms of .

 **(b)** A point P on this curve is equidistant from one of its asymptotes and the x-axis.

 Prove that, for all values m, P lies on the curve :

 **(a)** The asymptotes of the hyperbola are or .

 Compare this with the given symptotes , we have .

 Hence, .

 The hyperbola is therefore or .

 is on the hyperbola, therefore , we get = .

 The required equation of the hyperbola is .

 **(b)** Let the point P on the hyperbola be .

 The asymptotes or .

 The distance of P to the asymptotes is .

 The distance of P to x-axis is .

 Since is on , …(2)

 Substitute (1) in (2),

 Therefore is on the curve .

**4.** The tangents to the hyperbola at points A and B on the curve meet at point T.

 If M is the mid-point of AB, prove that TM passes through the center of the hyperbola.
 Prove that the product of the slopes of AB and TM is a constant.

 Let be two points on .

 Equation of tangent at A:

 Equation of tangent at B:

 Solve (1) and (2), .

 M =

 Let C(0,0) be the centre of the hyperbola,

 Slope of TC = , Slope of MC =

 Hence, Slope of TC = Slope of MC and TMC is a straight line.

 Therefore TM passes through the center of the hyperbola.

 Slope of TM =

 Slope of AB =

 Slope of TM x Slope of AB = (which is a constant).

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