**Hyperbola**

1. Prove that the point lies on the hyperbola .

Another point on this hyperbola is given by .

Find the equation of chord .

Deduce that the equation of the tangent to the hyperbola at is given by

This tangent intersects the x-axis at the point A and the y-axis at the point B.

Find the area of in terms of p.

Therefore the point lies on the curve .

Gradient of the chord =m =

The equation of chord :

The chord becomes the tangent to the curve at when Q = P, that is, q = p.

This tangent intersects the x-axis when or

and intersects the y-axis when or

Area of

**2.** Given four points , . A point P moves so that its distances are related by the equation: .

Show that the locus of P is a hyperbola and find the equations of its asymptotes.

Let ,

The locus of P is a hyperbola:

The asymptotes of the hyperbola are or .

**3.** **(a)** A curve of the form has asymptotes and passes through the point . Find the equation of the this curve in terms of .

**(b)** A point P on this curve is equidistant from one of its asymptotes and the x-axis.

Prove that, for all values m, P lies on the curve :

**(a)** The asymptotes of the hyperbola are or .

Compare this with the given symptotes , we have .

Hence, .

The hyperbola is therefore or .

is on the hyperbola, therefore , we get = .

The required equation of the hyperbola is .

**(b)** Let the point P on the hyperbola be .

The asymptotes or .

The distance of P to the asymptotes is .

The distance of P to x-axis is .

Since is on , …(2)

Substitute (1) in (2),

Therefore is on the curve .

**4.** The tangents to the hyperbola at points A and B on the curve meet at point T.

If M is the mid-point of AB, prove that TM passes through the center of the hyperbola.  
 Prove that the product of the slopes of AB and TM is a constant.

Let be two points on .

Equation of tangent at A:

Equation of tangent at B:

Solve (1) and (2), .

M =

Let C(0,0) be the centre of the hyperbola,

Slope of TC = , Slope of MC =

Hence, Slope of TC = Slope of MC and TMC is a straight line.

Therefore TM passes through the center of the hyperbola.

Slope of TM =

Slope of AB =

Slope of TM x Slope of AB = (which is a constant).

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